

# Sparsity-based super-resolution and phase-retrieval in waveguide arrays

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**Abstract:** We present a scheme for recovering the complex input field launched into a waveguide array, from partial measurements of its output intensity, given advance knowledge that the input is sparse. In spite of the fact that in general the inversion problem is ill-conditioned, we demonstrate experimentally and in simulations that the prior knowledge of sparsity helps overcome the loss of information. Our method is based on GESPAR, a recently proposed efficient phase retrieval algorithm. Possible applications include optical interconnects and quantum state tomography, and the ideas are extendable to other multiple input and multiple output (MIMO) communication schemes.

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## Introduction

The ever-increasing demand in processing speed is pushing to the limits of current inter-chip and inter-module bandwidth. In this context, especially important are on-chip optical interconnects, which hold the promise of increasing communication bandwidth to meet performance demands, while maintaining low power dissipation [1]. As in any information processing systems, power dissipation is a major issue also for on-chip systems, with interconnects taking an increasingly big part of the power, as process size gets smaller [2]. Clearly, minimizing the number of detectors in on-chip optical interconnects would be of major importance: it will save power, reduce system size, and decrease fabrication complexity.

Here, we propose and demonstrate an experimental proof of concept of an optical scheme that decreases the number of detectors needed to detect a given number of bits, assuming that the input is sparse. Such scenarios naturally arise for example in waveguide-based photonic devices, which usually serve as some kind of logic gate with only a small number of excited inputs at a given time. In addition, the proposed scheme allows recovery of phase information from intensity measurements – which may allow encoding information in the signal's optical phase while not requiring coherent detection. The use of fewer detectors is made possible by using ideas emerging from the recent work on sparsity-based sub-wavelength imaging [3–7], and sparse phase retrieval [8–10] which are related to an area in information processing referred to as *compressed sensing* (CS) [11–13]. CS exploits prior information that a sought signal is *sparse* – i.e. contains only a few nonzero values, in order to recover the full signal from a small number of measurements. Traditional CS techniques deal with measurements that are linear in the sought signal [13,14]. However, as was recently shown [5,8–10,15,16], sparsity-based concepts are useful also when the measurements are quadratic, such as in the problem of phase-retrieval: recovering the complex input field from measurements of intensity only. In this work, we utilize sparsity to recover the complex input field launched into a waveguide array from only partial measurements of its output intensity – without any phase measurement whatsoever. Since this yields a quadratic problem, we use a nonlinear (quadratic) extension of CS [5,9]. In particular, our algorithm uses the GESPAR method to our setting [9].

Before describing the current work, a short introduction of CS is instructive. In principle, CS [11,12] is a signal processing method aimed at reducing the number of measurements necessary to recover an input signal that goes through a non-invertible system. In CS, the information loss associated with the passage of the signal through the system is compensated by prior information that the input signal is sparse - i.e. contains only a few non-zero coefficients. The recovery is performed by finding the sparsest solution out of all possible solutions that are consistent with the measurements. Although the resulting problem is in

general difficult to solve, under appropriate conditions it can be shown that a unique sparse input can be found using computationally feasible algorithms [14]. An essential condition for CS recovery to work well is that each measurement needs to contain information, i.e. an impulse input signal should get 'smeared' as much as possible in the measurement domain.

### Scheme Description

The scheme in the current work consists of an array of evanescently-coupled single mode waveguides, where light couples from one waveguide to its neighbors. The propagation of light from plane  $z = 0$  to plane  $z = z_0$ , under tight-binding approximation, is described by the following coupling equation [17]:

$$i \frac{\partial}{\partial z} a_k + c(a_{k-1} + a_{k+1}) = 0 \quad (1)$$

where  $a_k$  is the complex modal amplitude in waveguide  $k$ . The impulse response of a one-dimensional array of coupled waveguides, namely, the solution to Eq. (1) with input  $a_0(z=0) = 1$ ,  $a_{k \neq 0}(z=0) = 0$  [17]:

$$h[k] = i^k J_k(2cz_0) \quad (2)$$

with  $h[k]$  representing the complex modal amplitude in waveguide  $k$ ,  $J_k$  being the  $k$ -order Bessel function, and  $c$  being the coupling coefficient, as shown in Fig. 1(a). The input signal is a discrete vector of complex mode-coefficients. As such, the signal propagation at every plane is given by convolution of the input signal with the impulse response at that plane, as shown in Fig. 1(b). We aim to recover this input signal from measurements of the intensity (only) at the output facet of the waveguide array, based on the advance knowledge that the input signal is sparse, i.e., the number of non-zero elements in the input signal is small. As known from the field of CS, sparsity-based signal recovery works best if the measurements are carried out in a basis that is least correlated with the basis in which the signal is sparse. It is therefore important to notice that, for sufficiently large value of  $cz_0$ , the input signal is smeared by the impulse response of the system (see Fig. 1, showing the spreading of the signal with propagation distance). This means that performing measurements at the output of a sufficiently long waveguide array facilitates the use of sparsity-based methods. As shown below, we exploit this feature for reconstructing the input information using fewer detectors, and without performing any phase measurements.

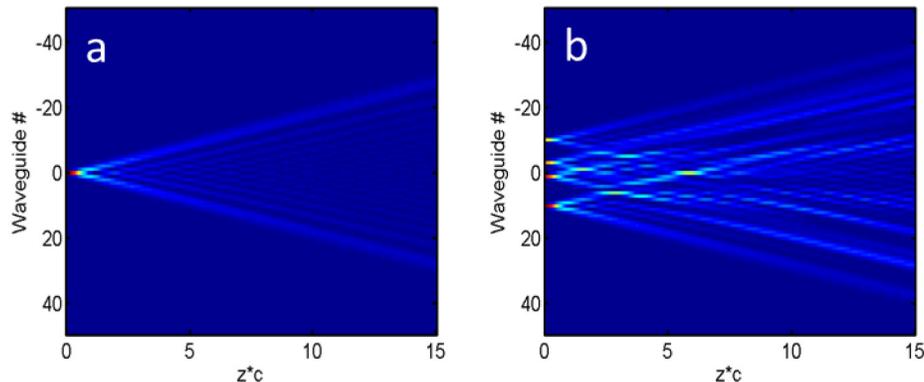


Fig. 1. (a) Impulse response for light propagating in a periodic waveguide array (intensity). The light is launched into the central waveguide (#0), and evolves such that the envelope corresponds to the absolute value squared of the expression in Eq. (1), from left to right. Each row represents an individual waveguide. (b) A different input signal would yield a correspondingly different propagation pattern.

The scheme consists of the following idea: Using a coupled waveguide array of  $n$  waveguides, we transmit a  $k$ -sparse input signal. That is, at  $z = 0$ , light is launched into  $k$  of the  $n$  waveguides with some unknown phase and amplitude distribution (where  $k \ll n$ , and  $k$  may also be unknown). The complex amplitudes of the light launched into the waveguides are the signal we wish to recover from intensity measurements at some other plane. The detection at plane  $z = z_0$  is done by  $m$  detectors (where  $m < n$ ), either by grouping several adjacent waveguides into a single detector and integrating over their output intensities, or simply by selecting a sub-group of detectors (e.g. for demultiplexing purposes). Note that without any prior knowledge on the input signal, this system is highly ill-posed due to the loss of phase information, and therefore practically non invertible in the presence of noise.

### Problem Formulation

To formulate the problem mathematically, we denote the sought input vector, containing the complex amplitudes of the  $n$  possible input waveguides by  $\mathbf{x} \in \mathbb{C}^n$ , and the measurement vector, containing  $m$  measured output intensities (with  $m < n$ ) by  $\mathbf{y} \in \mathbb{R}^m$ . Considering the case where the measurements simply consist of a subset  $M$  of size  $m$  out of all waveguide output intensities - the input-output relations of the discrete system can be formulated by the following set of equations:

$$y_i = |\mathbf{h}_i \cdot \mathbf{x}|^2 = \mathbf{x}^* \tilde{\mathbf{H}}_i \mathbf{x}, \quad i \in M \quad (3)$$

where  $\mathbf{h}_i \in \mathbb{C}^n$  is the  $i$ -th row of the Toeplitz matrix representing the system's transfer function, namely,  $H_{jk} = h[j-k]$ , where  $h[k]$  is the impulse response given by Eq. (2).  $\tilde{\mathbf{H}}_i \in \mathbb{R}^{n \times n}$  is a matrix defined by  $\tilde{\mathbf{H}}_i = \mathbf{h}_i^* \mathbf{h}_i$ , where  $*$  represents the conjugate transpose operator. Here we assumed that the measurements are taken far from the edges of the array, so that boundary effects are negligible.

Recovery of the unknown vector from the set of equations in Eq. (3) is an ill-posed problem: not only is the number of equations smaller than the unknowns ( $m < n$ ), but also the phase of the output signal is not measured – so the equations are in fact quadratic. Our problem is therefore to find a complex solution to  $m$  quadratic equations in  $n$  variables, with  $m < n$ , given only absolute value measurements. However, we have the prior information that our input signal is sparse. Relying on recent work [5,9] that deals with the similar problem of finding sparse solutions to the phase-retrieval problem (which constitutes a quadratic compressed sensing problem) – we employ the algorithm presented in [9]. GESPAR was originally intended to solve the sparse phase retrieval problem of recovering a sparse signal from measurements of its Fourier magnitude, but it can also be used to solve the more general sparse quadratic problem [9].

The physical system considered in the paper, consisting of many equally spaced identical waveguides – is shift invariant (the number of waveguides is practically infinite, as long as the propagation does not approach the last waveguide on either side). This is the reason for the Toeplitz form of the system's transfer matrix  $\mathbf{H}$ . However, this shift invariance is by no means a fundamental constraint – the solution method will be identical for a disordered system or a system with reflections from the boundaries. The only difference would be that an appropriate system matrix  $\mathbf{H}$  will be used.

### Solution method – GESPAR

GESPAR [9] is a method developed to solve the sparse phase retrieval problem, which is a special case of a sparse quadratic problem. In fact, it can be posed in the same way as Eq. (3), with an appropriately defined set of matrices  $\tilde{\mathbf{H}}_i$  [9]. The sparse quadratic problem that is actually solved is the following:

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \sum_{i=1}^m (y_i - \mathbf{x}^* \tilde{\mathbf{H}}_i \mathbf{x})^2 \quad (4)$$

subject to  $\|\mathbf{x}\|_0 \leq s$

where the parameter  $s$  is an upper limit to the number of excited waveguides. The cost function is denoted by  $f(\mathbf{x}) = \sum_{i=1}^m (y_i - \mathbf{x}^* \tilde{\mathbf{H}}_i \mathbf{x})^2$ .

In order to find a sparse solution  $\hat{\mathbf{x}}$  to Eq. (4), we use GESPAR with the set of matrices  $\tilde{\mathbf{H}}_i$  relevant to our problem. This in fact requires no modification to the formulation in [9] other than defining  $\tilde{\mathbf{H}}_i$  to correspond to our system. The stages in GESPAR are summarized below - for a more detailed description see [9]:

1. Select an initial random support for  $\mathbf{x}$ , namely the initial guessed locations of the excited waveguides.
2. Given the support, the problem defined by Eq. (4) reduces to a nonlinear least squares problem, which we solve by the damped-Gauss-Newton algorithm [18] that is commonly used for this type of problems. The damped-Gauss-Newton procedure produces an estimate  $\mathbf{x}$ .
3. Calculate the cost function  $f(\mathbf{x})$  along with its gradient around the current estimate.
4. Perform a local search by index swapping: replace an index  $i$  from the support containing a small absolute valued element with an index  $j$  containing a high absolute gradient value. Perform a damped-Gauss-Newton procedure given the new support, obtain  $\mathbf{x}_s$  and calculate the cost function  $f(\mathbf{x}_s)$ .
5. Update support: If the cost function is improved, i.e.  $f(\mathbf{x}_s) < f(\mathbf{x})$ , then the support is updated - i.e. the index  $i$  is removed from the support and the index  $j$  is added to it. Go to step 3. Otherwise, go to 4, until a predetermined number of swaps has been performed. If there is no possible improvement in the cost function by swapping - go to step 1.

The procedure stops after a predetermined number of swaps. The final answer is thresholded, zeroing all elements with amplitudes smaller than a parameter  $T$ .

### Experimental demonstration

We demonstrate the concept experimentally with the setup shown in Fig. 2. A collimated 808nm laser beam is illuminating a one-dimensional Spatial-Light-Modulator (128 pixel CRI SLM in phase and amplitude shaping mode, between two crossed polarizers). The output of the SLM is then imaged onto the input facet of a 25 mm long waveguide array (femtosecond laser micro-fabricated bulk glass waveguides [19]), containing 100 waveguides with 18 $\mu$ m inter-waveguide separation (producing a coupling constant of  $c = 1.44 \text{ cm}^{-1}$ , such that  $cz = 3.61$ ), with each waveguide mapped by 2 pixels in the SLM, so that the spatial modulation of the SLM determines the input signal. The light propagates through the array, and the output facet is imaged onto a CCD camera, where the light intensity distribution is measured.

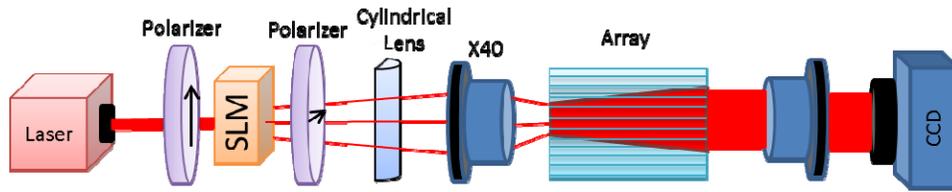


Fig. 2. Experimental scheme. The EM field emerging from the 1D SLM (working in amplitude mode between two crossed polarizers) is imaged onto the input facet of the waveguide array. The light undergoes discrete diffraction in the array. The output intensity is imaged onto a CCD camera.

The measured output intensity distribution is then used to algorithmically recover the input field – amplitude and phase. Note again that the measurement contains only partial information about the output field, namely, it contains no phase information, and its resolution is considerably lower than the minimum required for recovering the input field through traditional deconvolution methods. The intensity is first discretized into a discrete vector with each element corresponding to the output intensity of a single waveguide, and then the GESPAR algorithm [9] is used to recover the input field. The prior knowledge of sparsity is used since only a small number of waveguides are excited at the input of the array.

In the first experiment, shown in Fig. 3, we demonstrate recovery of a signal (amplitude and phase of the input field) from the complete set of available intensity measurements. Three waveguides are excited with different amplitudes and phases, and the light intensity and the output of 22 waveguides containing signal is measured (Fig. 3(a)). The signal is then discretized as a 1D vector – shown by the blue circles in Fig. 3(b). Using our technique, we are able to recover which waveguides were excited, at what amplitudes and with which relative phases (Fig. 3(c) and 3(d) respectively), by seeking the sparsest solution consistent with the measurements. Note that due to trivial ambiguity in an absolute-phase shift, the phase of the central waveguide is arbitrarily chosen as 0. The consistency with the measurements is shown by the green x's in Fig. 3(b) – corresponding to the light intensity at output facet that the recovered signal would yield. Comparing the recovered signal with the original input (green and blue circles in Fig. 3) clearly shows that our sparsity-based technique performs phase retrieval accurately.

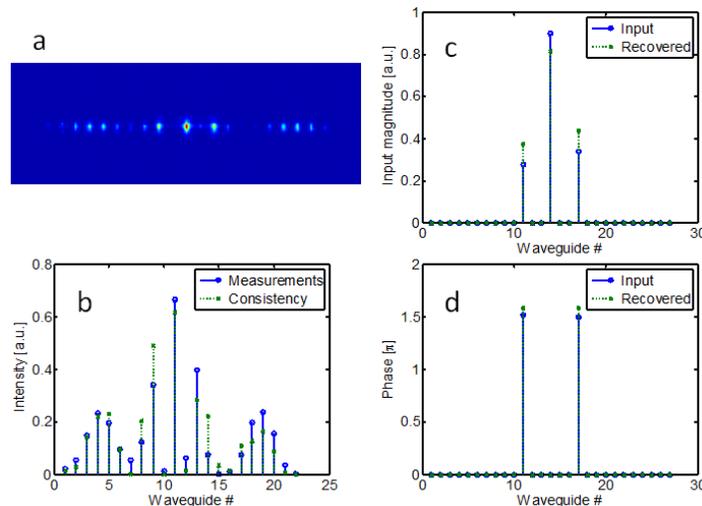


Fig. 3. Recovery from the complete set of measurements: (a) measured output intensity, (b) discretized measurements vector, and recovery results of (c) amplitude and (d) phase.

Next, we demonstrate phase retrieval from partial measurements, that is, combining phase retrieval with super-resolution. Conceptually, this experiment corresponds to a discrete version of recent work on sparsity-based sub-wavelength coherent diffractive imaging [6,7].

Figure 4 presents the results of experiments showing the recovery of a 3-sparse signal (Figs. 4(b) and 4(d) show the signal's amplitude and phase, respectively), from one half of the available measurements (optical intensities at the output of the waveguides), obtained by discarding every other measurement. Figure 4(e) shows the used part of the discretized measurements. The measured point-spread-function of the system is shown in Fig. 4(a), and the measured output intensities before discretization is shown in Fig. 4(c). Figure 4(f) demonstrates the super-resolution effect achieved by the method – where the non-measured intensities (every other waveguide) are in fact recovered with high accuracy. Here, waveguide 13 (the left input waveguide) is chosen as having phase = 0. The correspondence between the amplitude and phase of the recovered signal and the amplitude and phase of original input, displayed in Figs. 4(b), 4(d), demonstrates that our sparsity-based technique performs super-resolution with phase retrieval accurately.

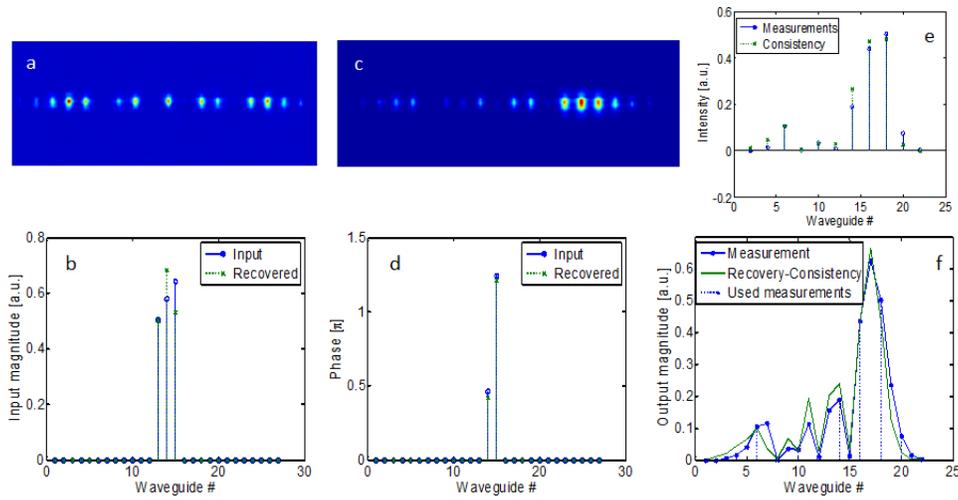


Fig. 4. Recovery from partial measurements. (a) Point spread function of the system. (b) Input and recovered amplitudes. (c) Measured output intensities. (d) Input and recovered phases. (e) The 11 measurements used and the consistency of the recovered signal at the system's output. (f) Super-resolution: accurate recovery of the non-measured output intensities (blue) using half of the output intensities (dashed blue).

## Numerical simulations

Having demonstrated the power of our sparsity-based method in phase-retrieval combined with super-resolution, it is important to study the generality of the method in terms of sparsity and noise. To do that, we perform numerical simulations, displayed in Fig. 5(a). In the simulations, we generate random complex input signals, which yield their corresponding partial output intensities using the transfer function of Eq. (1). We use GESPAR to recover the input signal from part of the measurements vector. Figure 5(a) shows the normalized

reconstruction error, defined as  $\frac{\|\mathbf{x} - \mathbf{x}_{rec}\|_2}{\|\mathbf{x}\|_2}$ , vs. the input signal's sparsity level  $k$ , for different

SNR levels. In these simulations, one half of the intensity measurements are used for reconstruction (20 out of 40), obtained by keeping only every other output waveguide intensity information. Each input signal is composed of  $k$  non-zero elements at randomly chosen locations with uniformly random phase in the range  $[0, 2\pi]$ , and uniformly distributed

magnitude in the range [1,2]. Each data point is the averaged result of 100 iterations. Figure 5(a) shows that good signal recovery (amplitude and phase recovery with error comparable to the noise level) of up to a sparsity level of 4. That is, our method facilitates super-resolution combined with phase retrieval when up to 4 waveguides are excited out of the possible 21 waveguides at the center of our array, given 20 intensity measurements and SNR of 30 (corresponding to ~3% of noise) and higher.

Another aspect we investigate numerically is the number of measurements necessary to recover a signal robustly for different sparsity levels. Figure 5(b) shows the probability to correctly recover the excited waveguides (the support of the signal), as a function of signal sparsity, for various levels of  $m$  – the number of measurements. The measurements are distributed evenly in the output plane, and the SNR is 40. As can be seen, the support recovery probability is very high (>95%) for sparsity of 4, using 24 measurements, out of the possible 40 over which the discrete diffraction pattern can extend given the propagation distance  $z$  and coupling constant  $c$ , which were taken such that  $cz = 4$ . If the sparsity is very low ( $k = 2$ ), 16 measurements empirically guarantee a robust recovery under the conditions of the simulation.

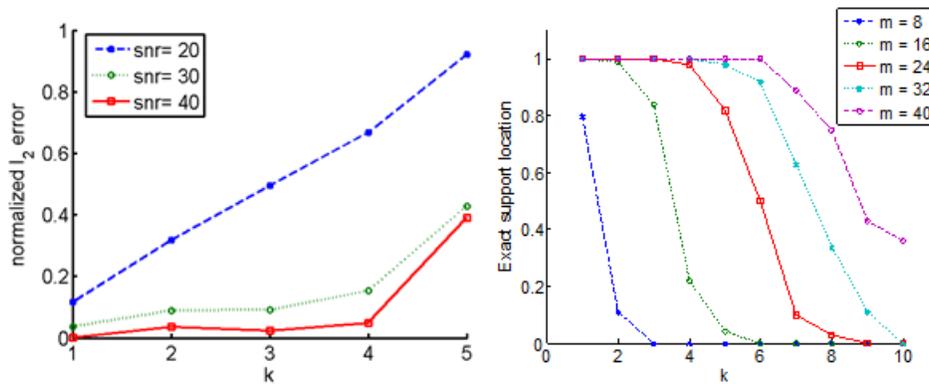


Fig. 5. (a) Reconstruction error vs. sparsity level ( $k$ ) for various SNR values. (b) Support location vs. sparsity levels for different values of  $m$  - number of evenly spaced intensity measurements

## Discussion

An interesting aspect to investigate is the uniqueness of the system, namely, under what circumstances does a set of measurements correspond to a unique input vector. More practically, in the presence of noise – when is it guaranteed that the recovered input vector is close to the true input? Without using the sparsity information, the system is very ill posed, and multiple, significantly different, input signals produce very similar output intensities. This can be easily verified by using GESPAR with a large sparsity parameter  $s$  and obtaining arbitrary non-sparse signals that are local minima of the problem in Eq. (4). However, as the simulations indicate, using the sparsity prior regularizes the inverse problem, allowing robust recovery of sparse signals with high probability.

In this context, we note that, while theoretical guarantees of robust sparse recovery from quadratic measurements do exist in the context of random measurements [16], the measurements in our system are structured, and therefore these guarantees do not directly apply. In other related recent work [20], uniqueness guarantees and recovery robustness of general phase-retrieval problems have been studied - but for general (non-sparse) signals, yielding sufficient uniqueness conditions on the measurement matrix, that are very hard to verify. However, in our case, the additional prior knowledge of signal sparsity may significantly improve these guarantees. Robustness guarantees for structured measurements of sparse signals are the subject of current work in progress, see for example [21].

One interesting property that the coupled waveguide system possesses is the following specific phase ambiguity: Denoting a certain excited waveguide at the input as waveguide 0 and setting its arbitrary phase to 0, then an identical output intensity will be attained by transforming the phase of all input waveguides according to:  $\phi_k \rightarrow k\pi - \phi_k$  where  $k$  is the waveguide index. This ambiguity is a mathematical property of the transfer function, and as such it is inherent in the system, similar to the mirroring and shifting ambiguities occurring in the Fourier phase retrieval problem. This ambiguity exists here even in the completely noiseless case.

Another aspect to be examined is the application of the scheme as a communication device. For example, when will it be beneficial to use the suggested scheme over simply using  $m$  uncoupled waveguides? On one hand, limiting the input signal to be sparse decreases the number of different possible transmittable signals. On the other hand, only by using coupling, one can recover the input phase information (In this sense the system can be seen as an interference measurement), so that a new degree of freedom to encode information is introduced. Whether using the sparse mode coupled system is beneficial will depend on the specific properties of the system, namely signal to noise ratio (SNR), coupling coefficient, etc., but a general framework of comparing certain aspects of the two approaches is presented.

To make a fair comparison, we allow multilevel intensity detection in both cases. Let us first consider a system of  $m$  uncoupled waveguides, with  $m$  intensity measurements. Multilevel detection implies non binary quantization of the input signal, and we denote by  $q$  the number of quantization levels that would allow a robust recovery (up to a pre-determined bit error rate) given the system's SNR. The number of different transmittable signals in the system is therefore  $n_{uc} = q^m$ .

Moving on to the coupled-waveguide system – we denote by  $q_a'$  and  $q_p'$  the number of quantization levels in a  $k$  sparse signal's amplitude and phase, respectively, that would allow its recovery using an array of  $n$  waveguides, given  $m$  intensity measurements. The number of possible signals is in this case is given by:

$$n_c = q_a'^k \cdot q_p'^k \cdot \binom{m}{k} = \frac{q_a'^k \cdot q_p'^k \cdot m!}{k!(m-k)!} \quad (5)$$

Using the coupled mode system will therefore be beneficial in the sense of the number of possible transmittable signals if:

$$\alpha = \frac{n_c}{n_{uc}} = \frac{q_a'^k \cdot q_p'^k \cdot m!}{q^m \cdot k!(m-k)!} > 1 \quad (6)$$

The above discussion is only partial, and many other considerations apply to an actual communication system, e.g. the process necessary to recover the input given the output. For example, in an actual communication application, the signal recovery would have to be very fast in order for the scheme to be useful. The exact performance details will depend on the implementation and on signal size and sparsity level, however, GESPAR has been shown to outperform existing phase retrieval algorithms in terms of computational speed as well as recovery performance [9]. For small enough signal sizes and sparsity the algorithm is very fast, requiring a small number of matrix inversions, the size of which is linear in the number of waveguides times the sparsity level.

Finally, it should be noted that having additional prior information on the input signal can improve the recovery performance. Adding prior information in addition to sparsity in order to improve recovery performance has been recently suggested in the context of image interpolation [22]. In our case, such additional information may be the total power in the system, a known amplitude relation between the input waveguides (e.g. all input waveguides have identical input amplitude - a binary input signal), or known phase relation - e.g. all input

waveguides have the same input phase. The latter can be easily incorporated into GESPAR, as it can be viewed as a real (rather than complex) input signal, which is the case dealt with in [9].

## Conclusion

In conclusion, we have demonstrated numerically and experimentally super-resolution phase-retrieval in an array of coupled waveguides. This is a generalization of sparsity based super resolution combined with phase retrieval [6] to a system with a transfer function different than free space propagation. This waveguide array serves as a model for a general system of optical interconnects, characterized by a specific transfer function. The work presented here has studied a one dimensional array, but naturally, interconnects can also have a higher dimensionality – depending on the number of nearest neighbors coupled to every specific site. From the recent paper on super-resolution coherent diffractive imaging, we conjecture that two dimensional interconnects would most probably also allow sparsity-based super-resolution phase-retrieval. It is most certainly interesting to explore phase retrieval in interconnects of a higher dimensionality. In a more general context, the method proposed here can be applied to various other multiple-input multiple-output (MIMO) communication schemes – for instance schemes based on multimode or multi-core fibers.

In addition, the technique can serve as the basis for super-resolution in quantum coincidence measurements in a waveguide array setting [23–25]. The ability to efficiently measure super-positions of states consisting of several photons is essential the characterization of quantum optical systems and computation units [26,27]. Such measurements are typically coincidence measurements. However, high-order coincidence measurements, which are required for the characterization of systems of more than two photons, are very hard to implement experimentally, and the number of different measurements required for the  $n$ -fold coincidence grows very fast with  $n$ . Sparsity may be used to reduce the number and the dimension of the required coincidence measurements, and a coupled waveguide array system can be suitable for an experimental demonstration of this concept [28].

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